P P SAVANI UNIVERSITY

Second Semester of B. Tech. Examination

May 2022

SESH1080 Linear Algebra & Calculus

28.05.2022, Saturday **Time: 10:00 a.m. To 12:30 p.m.** **Maximum Marks: 60**

Instructions:

1. The question paper comprises of two sections.
2. Section I and II must be attempted in same answer sheet.
3. Make suitable assumptions and draw neat figures wherever required.
4. Use of scientific calculator is allowed.

SECTION – I

Q - 1 Check the set of all 2×2 matrices of the form $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ with the standard matrix addition and multiplication is vector space or not. [05]

Q - 2 Sets of vectors $(1,1,2,1), (1,0,0,2), (4,6,8,6), (0,3,2,1)$ in R^4 are linearly dependent? [05]

Q - 3 Determine the dimension and a basis for the solution space of the system [05]

$$3x_1 + x_2 + x_3 + x_4 = 0$$
$$5x_1 - x_2 + x_3 - x_4 = 0$$

OR

Q - 3 Let $F: R^3 \to R^2$ be the linear map defined by $F(x, y, z) = (3x + 2y - 4z, x - 5y + 3z)$. Find matrix of F in the following bases of R^3 and R^2. $S = \{w_1, w_2, w_3\} = \{(1,1,1), (1,1,0), (1,0,0)\}$ and $S' = \{u_1, u_2\} = \{(1,3), (2,5)\}$. [05]

Q - 4 Let $T: R^3 \to R^3$ be the projection of a vector v into the xy-plane that is, $T(x, y, z) = (x, y, 0)$. Find kernel and range. [05]

Q - 5 Find a QR –decomposition of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. [05]

OR

Q - 5 Apply the Gram–Schmidt process to transform the basis vectors $u_1 = (1,1,1), u_2 = (0,1,1), u_3 = (0,0,1)$ into an orthogonal basis $\{v_1, v_2, v_3\}$, and then normalize the orthogonal basis vectors to obtain an orthonormal basis $\{q_1, q_2, q_3\}$. [05]

Q - 6 Find the least square solution of the linear system $Ax = b$ given by $x_1 + x_2 = 7$, $-x_1 + x_2 = 0, -x_1 + 2x_2 = -7$ and find the orthogonal projection of b on the column space of A. [05]

SECTION – II

Q - 1 Find all the local maxima, local minima, and saddle points of the function $f(x) = x^2 + xy + y^2 + 3x - 3y + 4$. [05]

OR

Q - 1 Find all the local maxima, local minima, and saddle points of the function $f(x) = x^2 + xy + 3x + 2y + 5$. [05]

Q - 2 Find $\frac{\partial w}{\partial v}$ when $u = 0, v = 0$ if $w = x^2 + \left(\frac{y}{x}\right), x = u - 2v + 1, y = 2u + v - 2$. [05]

Q - 3 [05]

(1) $\Gamma 1 =$ ____ (2) $\Gamma 0 =$ ____

(3) $\Gamma\left(\frac{1}{2}\right) =$ ____ (4) $\Gamma 2 =$ ____

(5) Symmetrical property of $B(m, n)$ is ____

Q - 4 Write Legendre's duplication formula and evaluate $\Gamma\left(\frac{5}{2}\right)$. [05]

Q - 5 Trace the hyplocycloid $x = a\cos^3 t, y = b\sin^3 t$. [10]

OR

Trace the cardioid $r = a(1 - \cos\theta)$.

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